Hall Ticket Number:

Code No. : 13165 FO

VASAVI COLLEGE OF ENGINEERING *(AUTONOMOUS)*, HYDERABAD

Accredited by NAAC with A++ Grade

B.E. III-Semester Backlog Examinations, Jan./Feb.-2024

Linear Algebra (OE-I)

Time: **3 hours** Max. Marks: **60**

*Note: Answer all questions from **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

Q. No.	Stem of the question	M	L	CO	PO
1.	Let $V = \mathbb{R}$. If addition and multiplication are defined by $\bar{a} \oplus \bar{b} = 2a + 2b$ and $k \odot \bar{a} = ka$ show that V is not a vector space.	2	1	1	1, 2, 12
2.	Determine whether $S = \left\{ \begin{bmatrix} 0 \\ y \end{bmatrix} / y \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^2$	2	2	1	1, 2, 12
3.	Define Linear Transformation between the two vector spaces. Show that the mapping $T: M_{mxn} \to M_{nxm}$ defined by $T(A) = A^t$ is a linear transformation.	2	1	2	1, 2, 12
4.	If $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y + z \\ -x + 5y + z \end{bmatrix}$ is a linear transformation. Find all the vectors mapped on to $\bar{0}$.	2	2	2	1, 2, 12
5.	Define Null Space and Range of a linear transformation.	2	1	3	1, 2, 12
6.	If linear operator $T: P_3 \to P_3$ is defined as $T(p(x)) = xp''(x)$. Determine whether the polynomial $p(x) = x^2 - 3x + 1$ is in N(T).	2	2	3	1, 2, 12
7.	Let $\bar{u} = \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$. Find the cosine of angle between the two vectors.	2	2	4	1, 2, 12
8.	Find the orthogonal projection of $\bar{u}$ onto $\bar{v}$ if $\bar{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$	2	2	4	1, 2, 12
9.	Find the coordinates of the vector $v = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ relative to the ordered basis $B = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$	2	2	1	1, 2, 12
10.	State sum of two linear transformations is a linear transformation.	2	1	2	1, 2, 12
	Part-B (5×8 = 40 Marks)				
11. a)	Show that the set of polynomials $S = \{x^2 + 2x + 1, x^2 + 2, x\}$ spans the vector space P_2.	4	1	1	1, 2, 12
b)	Find a basis of a vector space $V = \mathbb{R}^3$ that contain the vectors $S = \left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$	4	3	1	1, 2, 12
12. a)	If $T: P_2 \to P_2$ is a functions defined as $T(p(x)) = p''(x) - 2p'(x) + p(x)$. Is T a linear transformation?	4	1	2	1, 2, 12
b)	Define $T: P_3 \to \mathbb{R}^2$ by $T(ax^3 + bx^2 + cx + d) = \begin{bmatrix} -a - b + 1 \\ c + d \end{bmatrix}$. Let $\bar{u} = -x^3 + 2x^2 - x + 1$ and $\bar{v} = x^2 - 1$. Is $(\bar{u} + \bar{v}) = T(\bar{u}) + T(\bar{v})$?	4	2	2	1, 2, 12

Contd... 2

13. a)	If $T: P_2 \to P_2$ be defined by $T(ax^2 + bx + c) = ax^2 + (a - 2b)x + b$. Determine whether $p(x) = 2x^2 - 4x + 6$ is in the range of T.	4	2	3	1, 2, 12
b)	Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ x - y \end{bmatrix}$. If B is the standard ordered basis of $\mathbb{R}^2$ and $B' = \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \end{bmatrix}\right\}$ is ordered basis of $\mathbb{R}^2$ then find $[T]_B^{B'}$.	4	4	3	1, 2, 12
14. a)	Use the standard inner product on $\mathbb{R}^n$ to find $proj_{\bar{v}}\bar{u}$ given $\bar{u} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$	3	2	4	1, 2, 12
b)	If the inner product on P_2 is defined by $\langle p, q \rangle = \int_0^1 p(x)q(x)dx$, use the basis $B = \{x - 1, x + 2, x^2\}$ and the Gram-Schmidt process to find an orthonormal basis to P_2	5	4	4	1, 2, 12
15. a)	Find the transition matrix between the ordered basis $B_1 = \{1, x, x^2\}$ and $B_2 = \{x^2, 1, x\}$ given $[v]_{B_1} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$, find $[v]_{B_2}$	4	3	1	1, 2, 12
b)	If $T: M_{2x2} \to M_{2x2}$ is a linear operator and $T(e_{11}) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, T(e_{12}) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, T(e_{21}) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, T(e_{22}) = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$ then find $T\left(\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\right)$.	4	2	2	1, 2, 12
16. a)	Find a basis for the range of linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y + 3z \\ x + y + z \\ -x + 3y - 5z \end{bmatrix}$	4	3	3	1, 2, 12
b)	Let $V = C^{(0)}[a, b]$ with inner product $\langle f, g \rangle = \int_a^b f(x)g(x)dx$. Verify that the set of vectors $\{1, cosx, sinx\}$; $a = -\pi, b = \pi$ is orthogonal.	4	2	4	1, 2, 12
17.	Answer any ***two*** of the following:				
a)	Let $B_1 = \left\{\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}$ and $B_2 = \left\{\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}\right\}$ be two ordered basis of $\mathbb{R}^2$. Find $[I]_{B_1}^{B_2}$	4	4	1	1, 2, 12
b)	If $T: P_2 \to P_2$ is a linear operator and $T(1) = 1 + x, T(x) = 2 + x^2, T(x^2) = x - 3x^2$ then find $T(-3 + x - x^2)$.	4	3	2	1, 2, 12
c)	If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x \\ x + y \end{bmatrix}$ and $B = \left\{\begin{bmatrix} -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}, B' = \left\{\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}\right\}$. Find the matrix representation for T relative to the ordered bases B and ′ .	4	4	3	1, 2, 12

M : Marks; L: Bloom's Taxonomy Level; CO; Course Outcome; PO: Programme Outcome

i)	Blooms Taxonomy Level – 1	20%
ii)	Blooms Taxonomy Level – 2	40%
iii)	Blooms Taxonomy Level – 3 & 4	40%

*****